Determination of Planck's Constant from LED Knee Voltage and Wavelength

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Abstract

Light emitting diodes (LED's) in both the visible and infrared spectrum display a direct correlation between "knee" voltage and maximum intensity wavelength. Part of this proportionality constant is h, Planck's constant. Based on these relationships, data was collected for the wavelength and knee voltage of several LED's. The slope of the linear fit of the data was then used to solve for h. The linear fit, with a $\tilde{\chi}^2$ of 0.003, resulted in a value for h of 9.234 ± 0.004 $\times 10^{-}34$. This deviates significantly from the accepted value of h, with a T-value of 740. The relatively high precision of h while still being far outside of the acceptable range indicates a significant measurement bias, most likely in the reading of the knee voltage.

I. INTRODUCTION

LEDs are increasingly prevalent in our modern world and have many uses. However, in the physics lab, they are a useful demonstration of the quantized nature of light. Max Planck is credited with developing the theory of quantum mechanics, in which changes in atomic energy occur in discrete, or quantized, values. This theory of quantization relates the energy required to release or absorb a photon of light to the specific wavelength at which the light will be emitted. The energy and wavelength inversely are proportional and related by quantized multiples of a constant–Planck's constant–and the speed of light.[1] Since voltage is directly related to electrical energy, and LEDs emit light at a characteristic wavelength, the light emission of the LED must also be quantized, meaning there will be a specific energy, or voltage, at which the LED turns on. This energy is known as the knee voltage. However, there is some uncertainty in the method of determining where the knee voltage is actually read on an oscilloscope. Thus, this experiment is important because it provides a possible explanation for a discrepancy in accuracy between the methods of measuring knee voltage. The results of this experiment indicate that this is indeed an important factor, since h is known precisely, but not accurately, indicating a measurement bias that could be explained by the method of knee voltage measurement.

II. THEORY

Light emitting diodes–LED's–display a characteristic threshold voltage. This voltage is known as the knee voltage, V_k. Before this voltage, there is very little change in resistance through the LED. However, at V_k the resistance begins to increase exponentially with increasing voltage. This knee voltage is unique to each LED and varies inversely with the wavelength at which the LED emits at maximum intensity. This relationship between knee voltage and peak wavelength can be written as seen in Eq. (1) where

$$hf = e(V_k + V_0) \tag{1}$$

in which V_k is the knee voltage, V_0 is the initial constant that is unchanged for all LED's, e is the charge of an electron, h is Planck's constant, and f is the frequency of the light emitted by the LED. Values for h, c, and e come from the accepted values recorded by the National Institute of Standards and Technology (NIST).[2] As seen from Eq. (1), there is an intrinsic relationship between Planck's constant, the knee voltage, and the characteristics of the light emitted from the LED. In order to obtain this relationship in terms of wavelength rather than frequency, the value $\frac{c}{\lambda}$ can be substituted from f. Solving Eq. (1) to obtain its linearized form after this substitution gives

$$hf = \frac{hc}{\lambda} = e(V_k + V_0) \tag{2}$$

$$\frac{hc}{e\lambda} = V_k + V_0 \tag{3}$$

$$V_k = \frac{hc}{e}(\frac{1}{\lambda}) - V_0 \tag{4}$$

where V_k is the dependent variable, $\frac{hc}{e}$ is analogous to the slope of the linear model, $\frac{1}{\lambda}$ is the independent variable, and V_0 is analogous to the y-intercept of the linear model. Using this form of the relationship, the inverse relationship with Planck's constant, h, as part of the proportionality constant between λ and V_k can clearly be seen. This relationship is the basis of the procedure for this experiment. Knowing this relationship, data can be collected for λ and V_k for each LED and the data can be plotted against each other and fit to a linear regression model. The resulting slope, m, of this linear regression model contains within it the value of h, Planck's constant. Extracting this value from the linear model allows h to be solved for, resulting in

$$h = \frac{me}{c} \tag{5}$$

in which m is the slope extracted from the linear regression model fit to the data for λ vs. V_k. Since this linear regression model gives an average of the relationship between λ and V_k, along with its associated uncertainty, using this value of m from the linear fit produces the experimental value of h, Planck's constant. The accuracy of this linear fit can be tested using a $\tilde{\chi}^2$ method. To assess the suitability of the fit, the $\tilde{\chi}^2$ value can be calculated in two different ways. The first method is

$$\tilde{\chi}^2 = \frac{1}{d}\sum \frac{(O_k - E_k)^2}{E_k} \tag{6}$$

where d is the number of constraints (2 in the case of a linear model), O_k is the observed value at each bin and E_k is the expected value at each bin. However, the standard deviation, σ can also be used to determine the value of $\tilde{\chi}^2$, replacing E_k in the bottom portion of Eq. (6). This yields the equation

$$\tilde{\chi}^2 = \frac{1}{d}\sum \frac{(O_k - E_k)^2}{\sigma_k} \tag{7}$$

where σ is the standard deviation of the set of measurements, in this case wavelengths for each LED. The method of calculating $\tilde{\chi}^2$ is most often used in sets of data where measurements of a single value follow a standard distribution. It can, however, be adapted, as has been done in Eq. (7), to accommodate a set of data where each measurement is expected to follow a standard distribution. Thus, σ_k, the standard deviation of each measurement, replaces σ, the standard deviation for a single measurement, and the sum of deviations of O_k from E_k for each LED is divided by the standard deviation in the wavelengths for that LED, and the sum of these divided by the number of constraints gives the $\tilde{\chi}^2$ value. Comparing these values of $\tilde{\chi}^2$ obtained by Eq. (6) and (7) to determine whether they are both within the acceptable range of values for $\tilde{\chi}^2$ will add increased confidence in the suitability of the linear model.

III. METHODS AND MATERIALS

For this experiment, a circuit containing an LED and resistor was formed on a digital trainer, as shown in Fig.1 using red, yellow, green, aqua, blue LEDs along with two infrared LEDs of different wavelengths. One probe from the oscilloscope was used to measure the voltage across the entire circuit while the other probe was used to measure the voltage across the LED. The synthesis of these two voltage channels produces the voltage curve of the LED. As the amplitude of the current generated is increased, this voltage curve then produces the characteristic knee voltage at the point which the LED turns on.

This brings up the important distinction in the method by which V_k is measured. The first method of measuring V_k is to use the x-axis deviation adjustment mechanism on the oscilloscope to align origin with the point at which the voltage curve first begins to turn upwards. Doing so allows the voltage to be read from the deviation adjustment graphic. The second method of measuring V_k relies on the steep portion of the voltage curve rather than the flat section. This method extrapolates back to the x-axis from the portion of the steep curve for which the slope has become approximately constant. It can easily be seen that this second method of measuring V_k will produce a larger value than the first method. Ideally, the difference in these methods would be a moot point since the slope of the linear fit of the data is what is important, not whether it is shifted up or down (since V_k is plotted on the y-axis for the linear fit). Unfortunately, this simple one to one correlation in values between measurement methods does not exist. Rather, some of the LED voltage curves have a nearly perpendicular slope, resulting in values of V_k closer than normal to those measured by the first method while other LED voltage curves have a more gradual upturn in slope, producing a value of V_k that is farther than normal from that measured by the first method. For this reason, the first method of reading V_k was used, as it appeared that this method would lend itself to the greatest accuracy and repeatability. In order to account for as much difference as possible between the two measurement values, the scales on the oscilloscope were adjusted to make the voltage curve after the knee voltage as steep as possible. This did not, however, completely negate the effect of variance in rate of increase of the voltage curve around the knee voltage as discussed above.

The same circuit design on the digital trainer can be transferred to a darkened spectroscopy station in order to measure the relative light transmittance of each LED using the spectrophotometer setup shown in the second part of Fig. 1. The spectrophotometer measures the relative intensity of the light transmitted at each wavelength in the visible spectrum. From the recordings taken for each LED, the wavelength with the highest relative intensity of transmittance was used as the characteristic wavelength of that LED. However, since the spectrophotometer measured only in the visible spectrum, the wavelengths for the two infrared LEDs were determined using data provided with them. In this case, the two wavelengths were known, but needed to be assigned to the proper LED. Given the inverse relationship between V_k and λ, the larger wavelength was assigned to the infrared LED with the smaller V_k and the smaller wavelength was assigned to the LED with the larger V_k.

In either method of measuring V_k as well as in the measurement of λ, the instrument precision is so small that the main source of uncertainty comes from the variability in repeated measurement values. This variability is assumed to be random, following a standard Gaussian distribution. For this reason, five trials were recorded for measurements of V_k for each electrode using two different sets of LEDs. The mean and standard deviation of the five trials for V_k were then calculated and used as the y-values of the linear fit according to Eq. (4). The same determinations were also performed for the wavelength of each LED from the five trials of light transmittance for each LED. Of special note here, however, is that the uncertainty in wavelength for the infrared LEDs was assigned to be $\pm$ 10nm based on their provided data for the same reason that the wavelength for the infrared LEDs had to be assigned rather than measured.

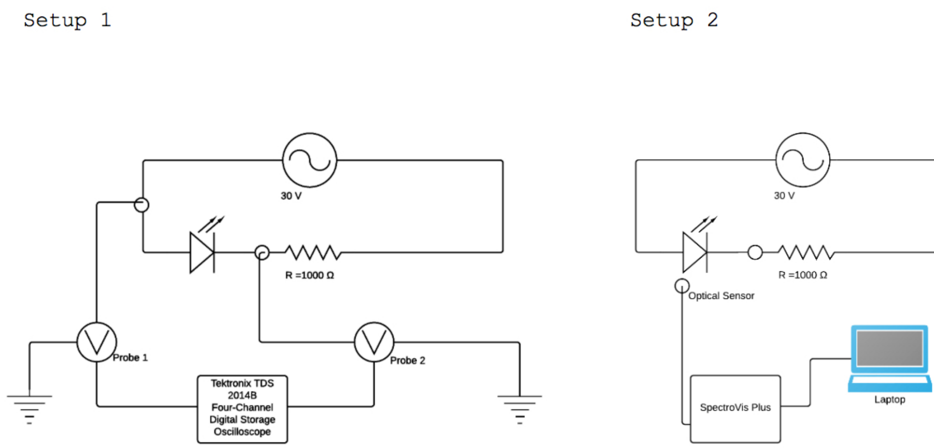

FIG. 1. This figure shows the circuitry schematic used in the experimental design along with the two measurement setups. In the first setup, Probe 1 measures the voltage across the entire circuit while Probe 2 measures the voltage across the LED. From these two values, the oscilloscope produces the LED voltage curve from which knee voltage is read. In the second setup, the spectrophotometer is used to measure the wavelength at which the LED transmits with maximum intensity.

IV. DATA PRESENTATION AND ANALYSIS

The data obtained from the plot of λ vs. V_k were plotted as seen in Fig. 2 along with their associated uncertainties. The associated value of $\tilde{\chi}^2$ obtained by Eq. (6) for the linear fit is 0.003, indicating an extremely high suitability of the linear model for the data. The $\tilde{\chi}^2$ value obtained using Eq. (7) was even smaller, confirming that the $\tilde{\chi}^2$ is indeed accurate. This provides strong evidence that we cannot reject this linear fit as an accurate model for the data. From the slope of the linear fit, using a Monte Carlo random error distribution model combined with the measurement error in both V_k and λ to determine the uncertainty in the slope, the experimental value of h was calculated using Eq.5 and the appropriate error propagation. This produced a value for h of $9.2234 \pm 0.004 \times 10^{-34}$ Js. With a t-statistic of 740, this value of h is clearly outside the acceptable range of deviation relative to the experimental uncertainty. Because measurement uncertainties and the resultant uncertainty in h are not unusually small, this large deviation from the accepted value indicates some form of measurement bias. Because the spectrophotometer measurements have been repeatedly

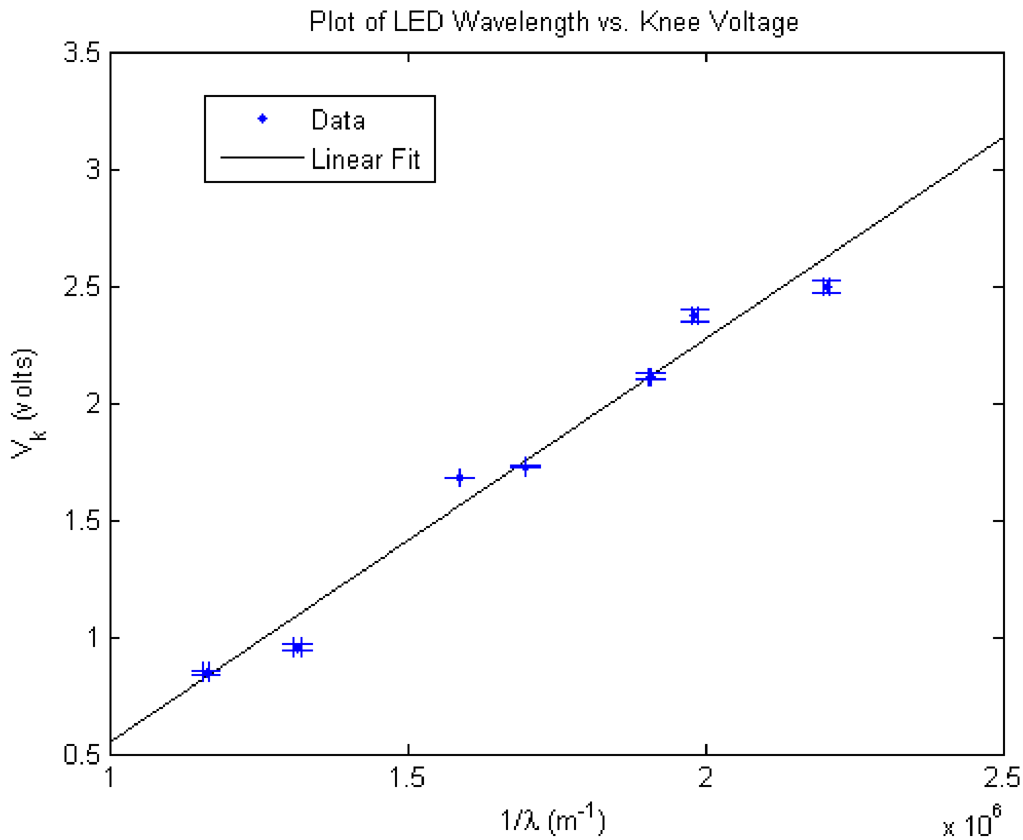

FIG. 2. This figure shows the linear fit of data for λ and V_k. The σ values for both λ and V_k are also included as the error bars on the data. The linear fit of the data has a $\tilde{\chi}^2$ value of 0.003, obtained using Eq. (6), indicating a high degree of suitability of the linear model for the data. This implies a high degree of confidence in slope determined according to Eq. (5), which produced a value of h of $9.2234 \pm 0.004 \times 10^{-34}$ Js.

used with accuracy in other experiment, and given the discrepancy in knee voltage measurement method, this measurement bias likely results from the measurement of V_k. Given the relationship between h and the slope of the linear fit, a smaller slope will produce a smaller h. An example of this can be obtained by taking linear fit of the last three data points, which results in a flatter slope and thus a smaller h. These three points were chosen for the adjusted fit because the values furthest below this adjusted line are those that had the most gradual upturn in their voltage curve. This more gradual curve means a larger discrepancy in V_k between measurement methods. This indicates that this adjusted curve may actually be closer to the true expected V_k values, indicating the possibility that due to the method of measurement of V_k, the first three to four data points may be too low, resulting in a slope, and thus h, that is too large. This indicates that the extrapolation method for measuring V_k may have been more accurate.

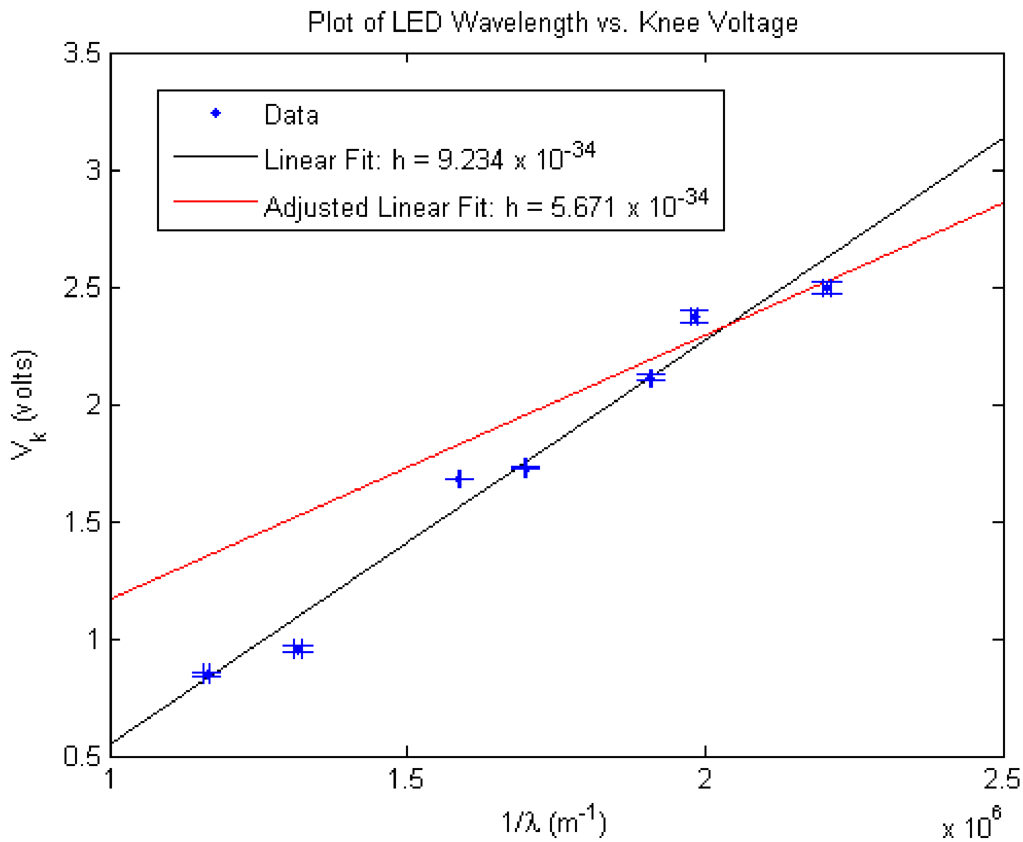

FIG. 3. This graph displays the plot of an adjusted linear model fit to the last three x-values compared to the original linear fit. The value of h determined for the adjusted linear fit is $5.671 \pm 0.004 \times 10^{-34}$ Js, which is significantly closer to the accepted value of h than the original linear fit. This indicates that there is a parallel correspondence between the two methods of measuring V_k

V. CONCLUSION

The purpose of this study was to experimentally determine the value of Planck's constant from the knee voltages and characteristic wavelengths of a set of LEDs and to determine whether a specific method of measuring the knee voltages of the LEDs would produce accurate values of h. To do so, the knee voltages, V_k, and the characteristic wavelengths, λ of a set of LEDs were measured using an oscilloscope and spectrophotometer, respectively. The data obtained were then modeled using a linear regression model to obtain a relationship between the two data analogous to Eq. (4). The $\tilde{\chi}^2$ value obtained using Eq. (6) for this linear fit is 0.003. The value of h obtained linear fit is $9.2234 \pm 0.004{\times}10^{-34}$ Js. The precision of h while still being significantly outside of the acceptable margin based on the t-statistic indicates a significant source of measurement bias. Since the spectrophotometer has been used to obtain accurate values for wavelength in many other experiments, the most likely source of the measurement bias is in the value of V_k determined by the oscilloscope. This, combined with the discrepancy in methods of knee voltage measurements provides strong evidence that the knee voltage measurement is the source of the experimental bias. The measurement bias resulting from the method of reading V_k from the oscilloscope is likely due to the fact that there is not a parallel correlation between the values of V_k for each LED. This arises from the fact that a more gradual upturn in the slope of the LED voltage curve will produce a value of V_k that has an even larger difference from the method used than for an LED voltage curve. This means that rather than producing a parallel shift in the plot of V_k vs. λ as would occur if the difference between V_k values was the same for each LED, it produces a greater shift in V_k values for which the upturn in the voltage curve is more gradual. This leads to the adjusted linear fit seen in Fig. 3 using the last three values. This adjusted linear fit produces a value of h that is significantly smaller and much closer to the accepted value of h than that of the original linear fit. This indicates that the correct slope of the linear fit is between the original and adjusted values. This is further supported by the fact that the first values of λ are those with the most gradual upturn in the graph of the knee voltage. All of this together indicates that the method of measuring the knee voltage by extrapolation is more accurate than the method chosen in this experiment.

These results are very pertinent to both future similar experiments and to A followup study would be needed to test this hypothesis. The best method for this study would be to collect multiple trials of data for V_k using the extrapolation method of reading the knee voltage from the oscilloscope. These data points could be analyzed in the same manner as the data for V_k in the current experiment and plotted as a second data set on top of the current V_k data using the existing values of λ. This would allow for a pairwise comparison of points to test whether the hypothesis of a nonparallel relationship between the two methods of measuring V_k. It would also allow for a statistical comparison of the slopes and resultant values of h along with a visual confirmation of the comparison.

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