Determination of Photopeak Energy Using a Scintillator Device

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Abstract

Using a sodium iodide scintillator detector, the Photoelectric Effect and Compton Scattering can be used to determine the original energy of the gamma ray. The purpose of this experiment is to test whether the pulse height from a sodium iodide scintillator detector is truly proportional to the energy deposited in the detector and to compare the accepted values for the Compton edge energy with actual measurements from the sodium iodide scintillator detector. We found that the measured values for Compton Edge matched the standard values within accepted uncertainties. Photopeak energy correlated to bin number in a linear fit with a $\tilde{\chi}^2$ value of 0.12. Three of our Compton Edge energies matched the accepted value to within less than 2σ, but two of our calculated values were outside of the acceptable margin based on the T-value.

I. INTRODUCTION

Because all elements have a unique radiation spectrum, not just in the visible spectrum but also in the rest of the electromagnetic spectrum, determining the energy of gamma ray radiation is also useful in identifying elements. Elements like Cesium-137, Sodium-22, and Copper-60 are all example of elements that emit gamma ray radiation at various energy levels. Gamma ray radiation exhibits two important phenomena upon interacting with a scintillator device–the Photoelectric Effect, in which electrons are excited by collision with a electromagnetic radiation and emit light when they return to a lower energy level,[1] and Compton Scattering, in which gamma rays are considered to "bounce" elastically off of electrons, transferring only a fraction of their energy–depending on how the gamma ray interacts with electrons it comes in contact with. These phenomena can be used to determine the photopeak energy for the incident gamma ray. The purpose of this experiment is to determine the accuracy of using a sodium iodide scintillator device by measuring the photopeak energy from gamma ray radiation from ^{22}Na, ^{60}Co, and ^{137}Cs and compare the values obtained for gamma ray energy to the accepted values form the National Nuclear Data Center.[2]

II. THEORY

When gamma rays interact with a NaI scintillator device, they can transfer their energy to electrons within the lattice of device by the Photoelectric Effect, Compton Scattering, or Pair Production. In the photoelectric effect, the incident gamma ray excites an electron out of the ionic lattice in the device, transferring all of its energy to the electron. This energy is transferred through other electrons in the device, which emit light as they return to lower energy levels. This results in a burst of light being emitted, which is detected by the scintillator device. The frequency of the light then determines the bin number the "photopeak" from this energy transfer and light emission is assigned to by the detector. The more often a single frequency of light is emitted, the higher the number of counts are in the respective bin. Each radioactive source has unique gamma ray energy at which it emits, thus generating a unique photopeak spectrum for each element. However, there is also a constant background radiation with its own unique photopeak spectrum which must be subtracted from spectrum of each element. Doing so gives rise to a radiation spectrum such as seen below in a later section. A Gaussian distribution around the photopeak is expected due to standard variations in measurements. If true, this Gaussian distribution can be fit to the Gaussian equation and used to find the center point of the sample data. Alternatively, the energy from the gamma ray may be only partly transferred the electron with which it collides in the phenomenon known as Compton Scattering. In Compton Scattering, the incident gamma ray is considered to scatter elastically at an angle θ off of the electron with which it collides. The energy transferred is at a minimum when the gamma ray deflects at zero degrees, indicating that the gamma ray did not deflect at all after passing the electron, thus transferring none of its energy. When the deflection angle is 180 °, which happens when the gamma ray collides head on with the electron and bounces straight back, the maximum amount of energy possible is transferred to the electron. The fraction of the gamma ray energy transferred to the electron then causes the electrons in the scintillator device to emit light, giving rise to an emission of light at a different wavelength than the photopeak despite the incident gamma having the same initial energy as those causing the emission at the photopeak wavelength. The incident gamma ray in the case of Compton Scattering then continues on with a lower energy than when it was initially emitted by the radioactive element. Using relativistic momentum, this energy difference can be calculated using Eq. (1) where

$$\frac{1}{E_f} - \frac{1}{E_i} = \frac{1 - cos\theta}{m_e c^2} \tag{1}$$

in which E_f is the energy after the gamma ray collides with the electron, E_i is the initial energy of the gamma ray, m_e is the mass of an electron, c is the speed of light, and θ is the reflection angle of the gamma ray. At the Compton Edge, θ is equal to 180 °. At this point E_f is equal to the Compton Edge energy, $E_{Compton}$. Solving for $E_{Compton}$ gives

$$\begin{aligned} \frac{1}{E_{Compton}} &= \frac{1 - cos(180\,\mathrm{deg})}{m_e c^2} + \frac{1}{E_i} && (2) \\ &= \frac{2}{m_e c^2} + \frac{1}{E_i} && (3) \\ E_{Compton} &= \frac{1}{\frac{2}{m_e c^2} + \frac{1}{E_i}} && (4) \end{aligned}$$

where E_i is the photopeak energy (i.e. the highest radiation energy recorded) obtained from the scintillator device due to the photoelectric effect. The area around each photopeak can then be fit to a Gaussian model. The model used for this data is of form

$$G(\sigma, x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-x_o)^2}{2\sigma^2}} \tag{5}$$

where σ is the standard deviation of the mean, x_o is the center value, and x is a data point. From this Gaussian fit, the center point, x_o can be extracted to give the bin number of the photopeak. This bin number can be plotted against the accepted values of the photopeak energies for each element. A linear fit can then be performed for this photopeak data. Because the bin numbers come from the experimental data, using them in the equation obtained from the linear fit of the accepted photopeak energies produces the values of the experimental photopeak energies. The experimental and accepted photopeak energies can then be used to calculate the Compton Edge energy using Eq.4. The error and uncertainty in these measurements, fits, and calculations can be propagated using standard methods to arrive at a final uncertainty. This final uncertainty will be used in the calculation of a t-statistic to determine the accuracy of our calculated Compton edge values.

III. METHODS AND MATERIALS

For this experiment, a NaI scintillator device was used as a photomultiplier for the Photoelectric Effect. The detector was connected by USB to a laptop running the Maestro program in order to sort photopeaks into bins. The radiation source was attached to a wooden block and placed 2cm from the end of the scintillator device, as shown in Fig.1. The software was then configured according to the settings on the NaI photomultiplier tube. Before radiation sources were tested, a five minute background radiation test was collected with all radioactive elements far away. This spectrum was then saved to be subtracted from the other radiation spectra during analysis. After collecting background radiation data, the ^{22}Na radioactive source was placed 2cm from the end of the scintillator device and data was collected until the main peak of the graph displayed in the Maestro interface reached 5,000 counts. After the data was saved the collection process was repeated for ^{60}Co and ^{137}Cs. Collection was also performed a second time for ^{60}Co to check for any significant drift in the photopeak data. After all data was collected each photopeak was fit to a Gaussian curve according to Eq.5, the center point was extracted, the photopeak energy and Compton Edge were calculated using Eq.4.

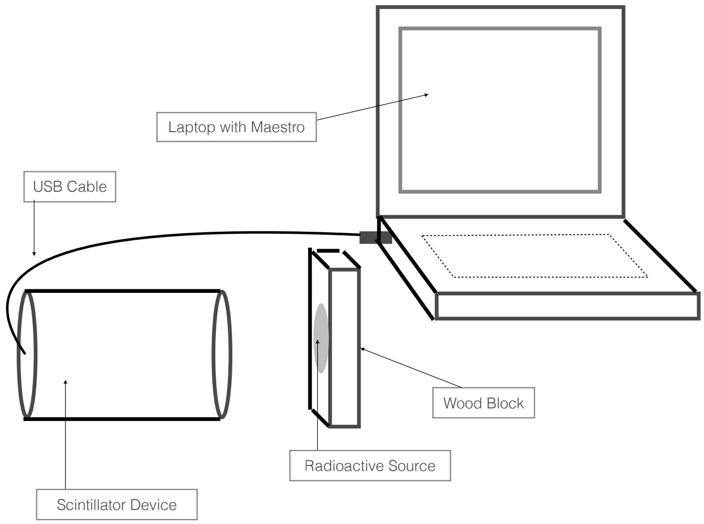

FIG. 1. This figure shows the apparatus used for the experiment. A NaI scintillator was attached by USB to a computer running the program Maestro to collect photopeak counts and place them in bins. The radiation source was attached with masking tape to a wooden block and placed 2cm from the end of the scintillator device.

IV. DATA PRESENTATION AND ANALYSIS

Photopeaks from the imported data were fit to a Gaussian curve. Fig.2 below shows the full spectrum of ^{22}Na along with the Gaussian fit of the second photopeak.

The raw data shows the two peaks of the Sodium spectrum (the two highest points) as well as the Compton Edge, seen in the slight rise to the left of the second photopeak. The Gaussian fit of the second photopeak is representative of all of the fits to the photopeak data. Using the Gaussian fit, the uncertainty for the x-axis (bins) as ± 1 bin, and the uncertainty in the y-axis as the square root of the number of counts for each specific bin, the calculated $\tilde{\chi}^2$ value for the fit is 0.29. With over 30 trials and two fit parameters, this $\tilde{\chi}^2$ represents a 100% match of the data to the Gaussian model. The drift test from ^{60}Co was also checked to ensure that the values from the collected data had not shifted during use. Using an uncertainty of 1 bin, there was no significant deviation of the drift test from the original data for ^{60}Co (t = 1σ). Bin values obtained from the Gaussian center were then plotted against the accepted value of gamma ray photopeak energy from the National Nuclear Data Center along with

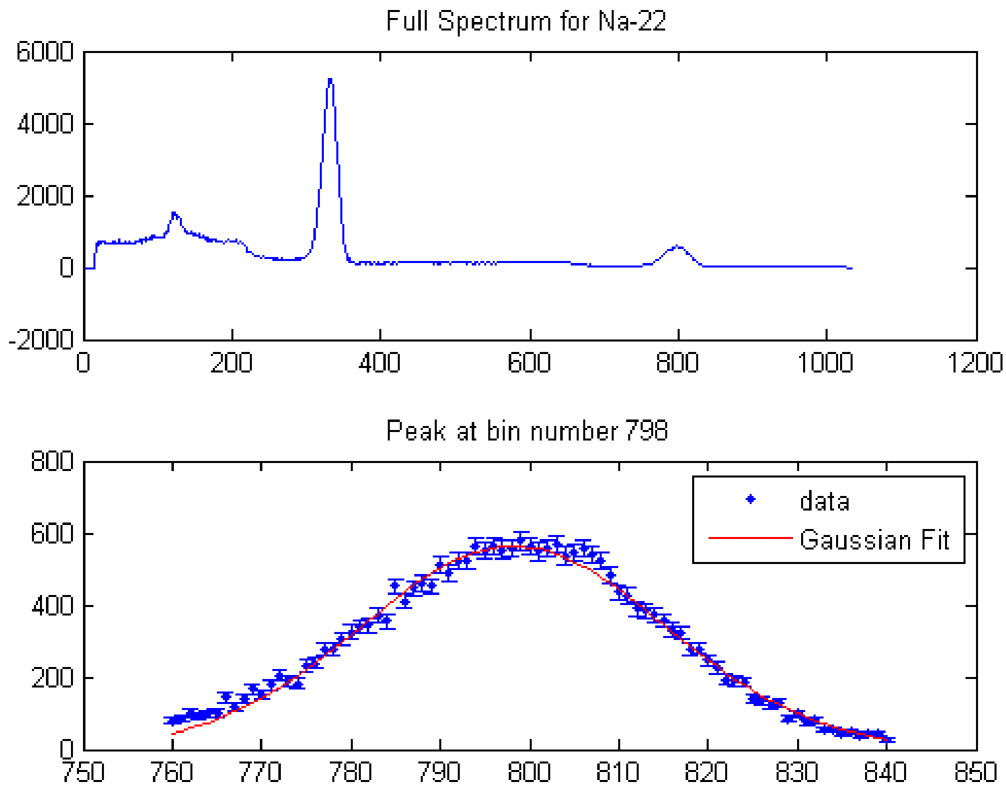

FIG. 2. This graph displays the plot of the full emission spectrum of ^{22}Na obtained from the Scintillator Device with a Gaussian fit of the second photopeak, where the x-axis is the bin number and the y-axis is in counts. Error bars on the data fit to the Gaussian curve are determined by the square root of the number of counts for each bin. The associated value of $\tilde{\chi}^2$ for the Gaussian fit is 0.29 indicating a high conformity of the data to the Gaussian model.

their associated uncertainties.[2] A linear fit of these data points and calculated values of the experimental photopeak energy values are plotted alongside the accepted values as shown in Fig.3.

The associated value of $\tilde{\chi}^2$ for the linear fit is 0.12, indicating a roughly 98% match of the data to the linear model, providing strong evidence that we cannot reject this linear fit as an accurate model for the data. From these photopeak energies and their associated uncertainties—either from the linear fit or from the standard values—the accepted and experimental values of the Compton Edge are calculated using Eq.4, yielding the results in Table I, along with the t-values for the deviation of the experimental values from the

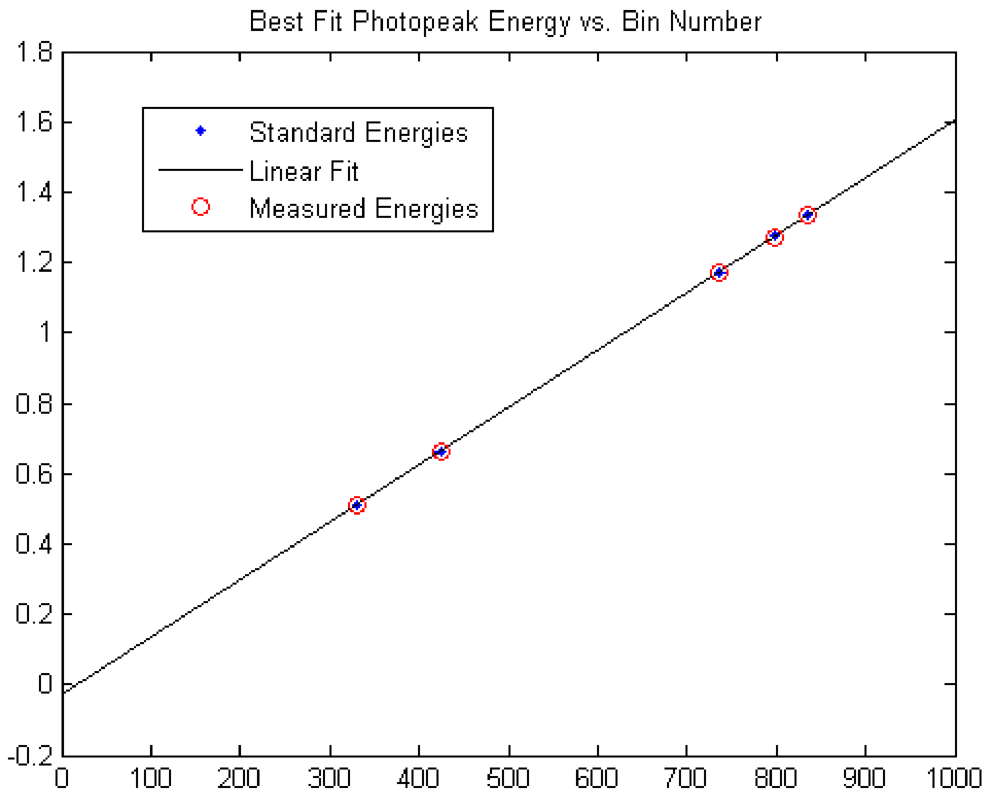

FIG. 3. This figure shows the linear fit of the experimental photopeak bins versus the accepted values for the photopeak energy where the x-axis is the bin number and the y-axis is in MeV. The σ of the accepted values are also included as the error bars on the data. The linear fit of the data has a $\tilde{\chi}^2$ value of 0.12, indicating a highly accurate match of the experimental prediction to the accepted energy value. Points on the line represent the experimental values of photopeak energy.

accepted for each Compton Edge energy. Early calculations using a method of rounding values to 3 decimal places resulted in T-values ranging from zero to 0.14. This would indicate a 0% to 11.13% chance that measurements of this accuracy would fall withing $t\sigma$ by random sampling alone. These values are due to the fact that all of our measured Compton edge values matched the accepted values to three decimal places except for the Cs peak. However, T-values of zero are highly suspicious, and upon further investigation, it was revealed that it was necessary to retain further decimal places in the calculation of Compton edge energy for both sets of data in order to see a meaningful deviation between our measured values and the accepted values. Doing so resulted in T-values ranging from

TABLE I. Compton Edge Energies

This table shows both the experimental and accepted values for the Compton Edge energy for each element. Experimental Compton Edge energy values and their uncertainties are calculated from the experimental photopeak energies and the accepted Compton Edge energies with uncertainties are calculated from the accepted photopeak energies

Element	Experimental $E_{Compton}$ (MeV)	Accepted $E_{Compton}$ (MeV)	t-value
^{22}Na Photopeak 1	0.1704902 ± 0.0000009	0.1704903 ± 0.0000001	0.097
^{22}Na Photopeak 2	0.2130246 ± 0.0000022	0.2130783 ± 0.0000070	5.838
^{60}Co Photopeak 1	0.2100430 ± 0.0000020	0.2100470 ± 0.0000030	0.797
^{60}Co Photopeak 2	0.2146512 ± 0.0000023	0.2146401 ± 0.0000040	1.768
^{137}Cs Photopeak 1	0.1845852 ± 0.0000011	0.1845071 ± 0.0000030	18.855

0.097 to 18.855. In this set of T-values, the second Na peak and the Cs peak stand out, as they are significantly outside of the acceptable margin. Because σ values are so small, the T-value increases rapidly when the measured value deviates from the accepted value in a lower decimal place. This indicates that while each of the Compton edge energy values were accurate to 3 decimal places, but become more inaccurate as decimal point precision increases. This higher discrepancy when retaining more decimal points of precision indicates that our measurements were accurate to the first few decimal places, but become increasingly inaccurate at more precise levels of measurement. The fact that only 2 elements stand out in this T-statistic is curious, however. This could be accounted for by the fact that these two peaks were both more sparse or varied in data points and thus the center value may have been less accurate, resulting in a less accurate Compton edge value.

V. CONCLUSION

The purpose of this experiment was to determine whether there truly was a direct correlation between bin number and photopeak energy and to confirm that photopeak and Compton Edge energies could be accurately measured using a NaI scintillator device. To do so, radiation spectrum data was collected from multiple radioactive elements. Each photopeak for each element was then fit to a Gaussian equation to confirm the standard distribution of the data and to extract a center point to use in the calculation of that bin's photopeak energy. Low $\tilde{\chi}^2$ values (around 0.29) for the Gaussian fit indicate a high degree of accuracy in assuming that the distribution of the photopeak data is indeed a standard Gaussian distribution. This also provides a high confidence and a small uncertainty in the value of the photopeak energy calculated from the data. The center bin values extracted from the Gaussian fit were then used as x-axis values for both the accepted values for photopeak energy for the photopeak of the corresponding element and photopeak energy calculated from the linear fit of the accepted values. A very low $\tilde{\chi}^2$ value for the linear fit (0.12) indicates a high degree of accuracy for the linear fit, providing strong evidence to confirm the direct relationship between bin number and photopeak energy. This low $\tilde{\chi}^2$ also provides a high confidence in the photopeak energy values calculated from the linear fit. The accepted and experimental values for the photopeak energy were then used to calculate the Compton Edge energy. The values for the Compton Edge energy from the accepted and experimental data were then compared using a t-test to evaluate the discrepancy between the accepted and experimental values. T-values for the first Na peak and both Co peaks fall well within the acceptable range, indicating a low discrepancy between our data and the accepted values and thus fairly accurate measurement and analysis techniques. This provides strong evidence to confirm the hypothesis that photopeak and Compton Edge energies can be accurately measured with a NaI scintillator device. However, with two of our five Compton edge values outside of the acceptable range at this precision level, this also provides evidence that would support the rejection of the hypothesis. Thus, we would conclude that there is strong evidence that photopeak energy and Compton edge energy can be measured accurately with a NaI scintillator device up to a precision of three decimal places, while higher decimal point precision is may lead to inaccurate results. In summary, low $\tilde{\chi}^2$ values for both the Gaussian and linear fit indicate that there is an extremely small likelihood that the data for photopeak energy could match the expected values as closely as they do by chance alone. Our T-values indicate a very small deviation of the experimental values from the accepted values for Compton Edge energy at smaller decimal point precision, but a larger deviation as precision increases above this point. Overall, this provides supports the acceptance of the data and of the NaI scintillator device as an accurate detector of pulse height for Photoelectric Effect and Compton

Scattering energies.

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[3] 1 J.R. Taylor, An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements (University Science Books, 1997).